www.artofproblemsolving.com/community/c583218
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– Paper 1

1 find all solutions, not necessarily positive integers for $(m^2+n)(m+n^2)=(m+n)^3$

2 P, Q, R and S are (distinct) points on a circle. PS is a diameter and QR is parallel to the diameter PS. PR and QS meet at A. Let O be the centre of the circle and let B be chosen so that the quadrilateral $POAB$ is a parallelogram. Prove that BQ = BP .

3 For each positive integer k, let a_k be the greatest integer not exceeding $\sqrt{k}$ and let b_k be the greatest integer not exceeding $\sqrt[3]{k}$. Calculate

$$\sum_{k=1}^{2003}(a_k-b_k).$$

4 Eight players, Ann, Bob, Con, Dot, Eve, Fay, Guy and Hal compete in a chess tournament. No pair plays together more than once and there is no group of five people in which each one plays against all of the other four.

(a) Write down an arrangement for a tournament of 24 games satisfying these conditions.

(b) Show that it is impossible to have a tournament of more than 24 games satisfying these conditions.

5 show that thee is no function f definedonthe positive real numbes such that :
$f(y)>(y-x)f(x)^2$

– Paper 2

1 If a,b,c are the sides of a triangle whose perimeter is equal to 2 then prove that:

a) $abc+\frac{28}{27}\geq ab+bc+ac$;

b) $abc+1<ab+bc+ac$

See also `http://www.mathlinks.ro/Forum/viewtopic.php?t=47939\&view=next` (problem 1)

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2 $ABCD$ is a quadrilateral. the feet of the perpendicular from D to AB, BC are P, Q respectively, and the feet of the perpendicular from B to AD, CD are R, S respectively. Show that if $\angle PSR = \angle SPQ$, then $PR = QS$.

3 Find all the (x,y) integer ,if $y^2 + 2y = x^4 + 20x^3 + 104x^2 + 40x + 2003$

4 Given real positive a,b , find the larget real c such that $c \leq max(ax + \frac{1}{ax}, bx + \frac{1}{bx})$ for all positive ral x.

There is a solution here,,,,
http://www.kalva.demon.co.uk/irish/soln/sol039.html
but im wondering if there is a better one .

Thank you.

5 (a) In how many ways can 1003 distinct integers be chosen from the set $\{1, 2, ..., 2003\}$ so that no two of the chosen integers di ffer by $10?$
(b) Show that there are $(3(5151) + 7(1700))101^7$ ways to choose 1002 distinct integers from the set $\{1, 2, ..., 2003\}$ so that no two of the chosen integers diff er by $10.$