JBMO Shortlist 2004

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–	Geometry
1	Two circles C_1 and C_2 intersect in points A and B. A circle C with center in A intersect C_1 in M and P and C_2 in N and Q so that N and Q are located on different sides wrt MP and $AB > AM$. Prove that $\angle MBQ = \angle NBP$.
2	Let E, F be two distinct points inside a parallelogram $ABCD$. Determine the maximum possible number of triangles having the same area with three vertices from points A, B, C, D, E, F.
3	Let ABC be a triangle inscribed in circle C. Circles C_1, C_2, C_3 are tangent internally with circle C in A_1, B_1, C_1 and tangent to sides $[BC], [CA], [AB]$ in points A_2, B_2, C_2 respectively, so that A, A_1 are on one side of BC and so on. Lines A_1A_2, B_1B_2 and C_1C_2 intersect the circle C for second time at points A', B' and C', respectively. If $M = BB' \cap CC'$, prove that $m(\angle MAA') = 90^\circ$.
4	Let ABC be a triangle with $m(\angle C) = 90^\circ$ and the points $D \in [AC], E \in [BC]$. Inside the triangle we construct the semicircles C_1, C_2, C_3, C_4 of diameters $[AC], [BC], [CD], [CE]$ and let $\{C, K\} = C_1 \cap C_2, \{C, M\} = C_3 \cap C_4, \{C, L\} = C_2 \cap C_3, \{C, N\} = C_1 \cap C_4$. Show that points K, L, M, N are concyclic.
5	Let ABC be an isosceles triangle with $AC = BC$, let M be the midpoint of its side AC, and let Z be the line through C perpendicular to AB. The circle through the points B, C, and M intersects the line Z at the points C and Q. Find the radius of the circumcircle of the triangle ABC in terms of $m = CQ$.