JBMO Shortlist 2005

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–	Geometry
1	Let ABC be an acute-angled triangle inscribed in a circle k. It is given that the tangent from A to the circle meets the line BC at point P. Let M be the midpoint of the line segment AP and R be the second intersection point of the circle k with the line BM. The line PR meets again the circle k at point S different from R. Prove that the lines AP and CS are parallel.
2	Let $ABCD$ be an isosceles trapezoid with $AB = AD = BC, AB//CD, AB > CD$. Let $E = AC \cap BD$ and N symmetric to B wrt AC. Prove that the quadrilateral $ANDE$ is cyclic.
3	Let $ABCDEF$ be a regular hexagon and $M \in (DE)$, $N \in (CD)$ such that $m(\widehat{AMN}) = 90^\circ$ and $AN = CM\sqrt{2}$. Find the value of $\frac{DM}{ME}$.
4	Let ABC be an isosceles triangle $(AB = AC)$ so that $\angle A < 2\angle B$. Let D, Z points on the extension of height AM so that $\angle CBD = \angle A$ and $\angle ZBA = 90^\circ$. Let E the orthogonal projection of M on height BF, and let K the orthogonal projection of Z on AE. Prove that $\angle KDZ = \angle KDB = \angle KZB$.
5	Let O be the center of the concentric circles C_1, C_2 of radii 3 and 5 respectively. Let $A \in C_1, B \in C_2$ and C point so that triangle ABC is equilateral. Find the maximum length of $[OC]$.
6	Let C_1, C_2 be two circles intersecting at points A, P with centers O, K respectively. Let B, C be the symmetric of A wrt O, K in circles C_1, C_2 respectively. A random line passing through A intersects circles C_1, C_2 at D, E respectively. Prove that the center of circumcircle of triangle DEP lies on the circumcircle of triangle OKP.
7	Let $ABCD$ be a parallelogram. $P \in (CD), Q \in (AB)$, $M = AP \cap DQ$, $N = BP \cap CQ$, $K = MN \cap AD$, $L = MN \cap BC$. Prove that $BL = DK$.