**Iran Geometry Olympiad 2015**

www.artofproblemsolving.com/community/c593235

by Snakes, parmenides51, MRF2017, pedestrilo14, acupofmath, mofumofu

**–** Elementary

---

**1** We have four wooden triangles with sides $3, 4, 5$ centimeters. How many convex polygons can we make by all of these triangles? (Just draw the polygons without any proof)

A convex polygon is a polygon which all of it's angles are less than $180^o$ and there isn't any hole in it. For example:
`https://1.bp.blogspot.com/-JgvF_B-uRag/W1R4f4AXxTI/AAAAAAAAIzc/Fo3qu3pxXcoElk01RTYJYZNwj`
`s640/igo%2B2015.el1.png`

---

**2** Let $ABC$ be a triangle with $\angle A = 60^o$. The points $M, N, K$ lie on $BC, AC, AB$ respectively such that $BK = KM = MN = NC$. If $AN = 2AK$, find the values of $\angle B$ and $\angle C$.

by Mahdi Etesami Fard

---

**3** In the figure below, we know that $AB = CD$ and $BC = 2AD$. Prove that $\angle BAD = 30^o$.
`https://3.bp.blogspot.com/-IXi_8jSwzlU/W1R5IydV5uI/AAAAAAAAIzo/2sREnDEnLH8R9zmAZLCkVCGeMa`
`s400/IGO%2B2015.el3.png`

---

**4** In rectangle $ABCD$, the points $M, N, P, Q$ lie on $AB$, $BC$, $CD$, $DA$ respectively such that the area of triangles $AQM$, $BMN$, $CNP$, $DPQ$ are equal. Prove that the quadrilateral $MNPQ$ is parallelogram.

by Mahdi Etesami Fard

---

**5** Do there exist $6$ circles in the plane such that every circle passes through centers of exactly $3$ other circles?

by Morteza Saghafian

---

**–** Medium

---

**1** Given a circle and Points $P, B, A$ on it.Point $Q$ is Interior of this circle such that:
$1)\ \angle PAQ = 90.$
$2) PQ = BQ.$
Prove that $\angle AQB - \angle PQA = \overset{\frown}{AB}.$

proposed by Davoud Vakili, Iran.

---

**2** In acute-angled triangle $ABC$, $BH$ is the altitude of the vertex $B$. The points $D$ and $E$ are midpoints of $AB$ and $AC$ respectively. Suppose that $F$ be the reflection of $H$ with respect to $ED$. Prove that the line $BF$ passes through circumcenter of $ABC$.

by Davood Vakili

---

**3** In triangle $ABC$ ,$M,N,K$ are midpoints of sides $BC,AC,AB$,respectively.Construct two semicircles with diameter $AB,AC$ outside of triangle $ABC$.$MK,MN$ intersect with semicircles in $X,Y$.The tangents to semicircles at $X,Y$ intersect at point $Z$.Prove that $AZ\perp BC$.(Mehdi E'tesami Fard)

---

**4** Same as Advanced P2

---

**5** a) Do there exist 5 circles in the plane such that every circle passes through centers of exactly 3 circles?
b) Do there exist 6 circles in the plane such that every circle passes through centers of exactly 3 circles?

---

**–** Advanced

---

**1** let $w_1$ and $w_2$ two circles such that $w_1\cap w_2=\{A,B\}$

let $X$ a point on $w_2$ and $Y$ on $w_1$ such that $BY\perp BX$

suppose that $O$ is the center of $w_1$ and $X'=w_2\cap OX$

now if $K=w_2\cap X'Y$ prove $X$ is the midpoint of arc $AK$

---

**2** let $ABC$ an equilateral triangle with circum circle $w$

let $P$ a point on arc $BC$ ( point $A$ is on the other side )

pass a tangent line $d$ through point $P$ such that $P\cap AB=F$ and $AC\cap d=L$

let $O$ the center of the circle $w$

prove that $\angle LOF>90^0$

---

**3** let $H$ the orthocenter of the triangle $ABC$

pass two lines $l_1$ and $l_2$ through $H$ such that $l_1\perp l_2$

we have $l_1\cap BC=D$ and $l_1\cap AB=Z$

also $l_2\cap BC=E$ and $l_2\cap AC=X$ like this picture

pass a line $d_1$ through $D$ parallel to $AC$ and another line $d_2$ through $E$ parallel to $AB$

let $d_1\cap d_2=Y$

prove $X$ , $Y$ and $Z$ are on a same line

---

**4** In triangle $ABC$, we draw the circle with center $A$ and radius $AB$. This circle intersects $AC$ at two points. Also we draw the circle with center $A$ and radius $AC$ and this circle intersects $AB$ at two points. Denote these four points by $A_1, A_2, A_3, A_4$. Find the points $B_1, B_2, B_3, B_4$ and $C_1, C_2, C_3, C_4$ similarly. Suppose that these $12$ points lie on two circles. Prove that the triangle $ABC$ is isosceles.

---

**5** we have a triangle $ABC$ and make rectangles $ABA_1B_2$ , $BCB_1C_2$ and $CAC_1A_2$ out of it.

then pass a line through $A_2$ perpendicular to $C_1A_2$ and pass another line through $A_1$ perpendicular to $A_1B_2$.

let $A'$ the common point of this two lines.
like this we make $B'$ and $C'$.

prove $AA'$ , $BB'$ and $CC'$ intersect each other in a same point.

---