Greece Team Selection Test 2015

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1	Solve in positive integers the following equation; $xy(x+y-10)-3x^2-2y^2+21x+16y=60$
2	Consider 111 distinct points which lie on or in the internal of a circle with radius 1.Prove that there are at least 1998 segments formed by these points with length $\leq \sqrt{3}$
3	Let ABC be an acute triangle with $AB<AC<BC$ inscribed in circle $c(O,R)$.The excircle (c_A) has center I and touches the sides BC, AC, AB of the triangle ABC at D, E, Z respectively.AI cuts (c) at point M and the circumcircle (c_1) of triangle AZE cuts (c) at K.The circumcircle (c_2) of the triangle OKM cuts (c_1) at point N.Prove that the point of intersection of the lines AN, KI lies on (c).
4	Find all functions $f:\mathbb{R}\to\mathbb{R}$ which satisfy $yf(x)+f(y)\geq f(xy)$