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by InCtrl

1 A chess tournament had 10 participants. Each round, the participants split into pairs, and each pair played a game. In total, each participant played with every other participant exactly once, and in at least half of the games both the players were from the same town. Prove that during each round there was a game played by two participants from the same town.

(Boris Frenkin)

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3 From given positive numbers, the following infinite sequence is defined: a_1 is the sum of all original numbers, a_2 is the sum of the squares of all original numbers, a_3 is the sum of the cubes of all original numbers, and so on (a_k is the sum of the k-th powers of all original numbers).
a) Can it happen that $a_1 > a_2 > a_3 > a_4 > a_5$ and $a_5 < a_6 < a_7 < \ldots$? (4 points)
b) Can it happen that $a_1 < a_2 < a_3 < a_4 < a_5$ and $a_5 > a_6 > a_7 > \ldots$? (4 points)

(Alexey Tolpygo)

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4 All the sides of the convex hexagon $ABCDEF$ are equal. In addition, $AD = BE = CF$. Prove that a circle can be inscribed into this hexagon.

(Boyan Obukhov)

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5 There is a set of control weights, each of them weighs a non-integer number of grams. Any integer weight from 1 g to 40 g can be balanced by some of these weights (the control weights are on one balance pan, and the measured weight on the other pan).What is the least possible number of the control weights?

(Alexandr Shapovalov)

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6 A grasshopper can jump along a checkered strip for $8, 9$ or 10 cells in any direction. A natural number n is called jumpable if the grasshopper can start from some cell of a strip of length n and visit every cell exactly once. Find at least one non-jumpable number $n > 50$.

(Egor Bakaev)

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7 1×2 dominoes are placed on an 8×8 chessboard without overlapping. They may partially stick out from the chessboard but the center of each domino must be strictly inside the chessboard (not on its border). Place on the chessboard in such a way:
a) at least 40 dominoes, (3 points)

b) at least 41 dominoes, (3 points)
c) more than 41 dominoes. (6 points)

(Mikhail Evdokimov)