

Turkey Junior National Olympiad 2017

www.artofproblemsolving.com/community/c602927

by CinarArslan

1 Find all triplets of positive integers (a, b, c) for which the number $3^a + 3^b + 3^c$ is a perfect square.

2 In a chess festival that is held in a school with 2017 students, each pair of students played at most one match versus each other. In the end, it is seen that for any pair of students which have played a match versus each other, at least one of them has played at most 22 matches. What is the maximum possible number of matches in this event?

3 In a convex quadrilateral $ABCD$ whose diagonals intersect at point E , the equalities

$$\frac{|AB|}{|CD|} = \frac{|BC|}{|AD|} = \sqrt{\frac{|BE|}{|ED|}}$$

hold. Prove that $ABCD$ is either a parallelogram or a cyclic quadrilateral

4 If real numbers $a > b > 1$ satisfy the inequality

$$(ab + 1)^2 + (a + b)^2 \leq 2(a + b)(a^2 - ab + b^2 + 1)$$

what is the minimum possible value of $\frac{\sqrt{a-b}}{b-1}$
