www.artofproblemsolving.com/community/c628064
by blue8931

1 Compute the value of $\frac{2^0+1^8}{2-0+1-8}$.

(A) $-\frac{3}{5}$ **(B)** $-\frac{2}{5}$ **(C)** $-\frac{1}{5}$ **(D)** $\frac{1}{5}$ **(E)** $\frac{2}{5}$

---

2 If three apples and one banana cost $\$9$ and six apples and five bananas cost $\$27$, then how many bananas can be purchased with $\$60$?

(A) 10 **(B)** 12 **(C)** 15 **(D)** 20 **(E)** 30

---

3 John wants to buy a jacket, and he has two options for coupons. In option A, John gets a 25% discount off of the original price, and in option B, John gets a $\$10$ discount off of the original price. If the original price of the jacket is $\$50$, then which option gives a larger discount, and by how much? Assume that there is no tax involved.

(A) Option A, by $\$2.50$

(B) Option A, by $\$1.25$

(C) Both options give the same discount

(D) Option B, by $\$1.25$

(E) Option B, by $\$2.50$

---

4 Max has an unlimited supply of both water and fruit punch. He pours 4 cups of water and 4 cups of fruit punch into a large water cooler. How many more cups of fruit punch does Max need to add so that the mixture consists of 75% fruit punch?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

---

5 What is the value of the following expression: $2017 \cdot 2019 - 2014 \cdot 2022$?

(A) 5 **(B)** 15 **(C)** 25 **(D)** 35 **(E)** 45

---

6 Tommy has three indistinguishable white cards and two indistinguishable black cards. How many ways can he place these five cards in a row so that the two black cards are never next to one another?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

---

7 Brent has a list that consists of the first 50 positive integers. He then deletes all of the numbers that are multiples of two from his list. After doing this, he then deletes all of the remaining numbers that are multiples of three. How many numbers are left in his list after doing these two deletions?

(A) 15 **(B)** 16 **(C)** 17 **(D)** 18 **(E)** 19

---

8 Alice, Ben, and Cam meet at the park on January 1st, 2018, a Monday. Alice visits the park once every 4 days (so the next time she visits the park will be on Friday, January 5th), Ben visits the park once every 6 days, and Cam visits the park once every 10 days. On what day of the week will all three of them meet at the park again?

(A) Tuesday **(B)** Wednesday **(C)** Thursday **(D)** Friday

(E) Saturday

---

9 How many three-digit numbers have the property that their digits form a non-constant geometric sequence when read from left to right?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 6 **(E)** 8

---

10 Triangle ABC has a right angle at B, and side lengths $AB = 3, BC = 4, AC = 5$. Points D and E are drawn on BC such that $DE = 2$ and $BD < BE$. Lines l and m, which are both parallel to AB, pass through D and E and intersect AC at D' and E', respectively. Given that the area of $DD'E'E$ is 2, determine the length of BD.

(A) 1 **(B)** $\frac{4}{3}$ **(C)** $\frac{3}{2}$ **(D)** $\frac{5}{3}$ **(E)** 2

---

11 Alex and Ben are running a 100-meter race. If Alex gets a 40 meter headstart (that is, Alex runs 60 meters and Ben runs 100 meters), then he finishes 2 seconds before Ben. However, if Ben gets a 40 meter headstart, then he finishes 6 seconds before Alex. If Alex and Ben were to both run a full 100 meter race, then in order for Alex and Ben to both finish the race at the same time, Alex should receive a k second headstart. What is k?

(A) $\frac{5}{2}$ **(B)** $\frac{11}{4}$ **(C)** 3 **(D)** $\frac{13}{4}$ **(E)** $\frac{7}{2}$

---

12 There exists a unique positive integer n less than 100 such that n has six factors, $n + 2$ has eight factors, and $n + 4$ has ten factors. In what interval does n lie?

(A) $[1, 19]$ **(B)** $[20, 39]$ **(C)** $[40, 59]$ **(D)** $[60, 79]$ **(E)** $[80, 99]$

---

13 A fair, six-sided die is rolled until the running sum is a positive multiple of 3. For example, if the die is rolled three times and lands on $4, 1, 2$, then the running sum is 4 after the first roll, 5 after the second roll, and 7 after the third roll. What is the probability that the die is rolled 2 or fewer times?

(A) $\frac{4}{9}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{5}{9}$ **(D)** $\frac{11}{18}$ **(E)** $\frac{2}{3}$

---

14 A sequence $a_1, a_2, a_3, a_4, \cdots, a_n$ has the property that $a_n + a_{n+1} = 2^n$ for all $n \geq 1$. If $a_1 = 1$, then what is the remainder when a_{2018} is divided by 10?

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

---

15 The polynomial $x^3 + 6x^2 + 4x + 2$ has roots r, s, and t. The polynomial $x^3 + ax^2 + bx + c$ has roots $r-1, s-1$, and $t-1$. What is $a+b+c$?

(A) 39 **(B)** 40 **(C)** 41 **(D)** 42 **(E)** 43

---

16 What is the smallest possible value of n such that value of the expression $\left\lfloor \frac{1}{24} \right\rfloor + \left\lfloor \frac{2}{24} \right\rfloor + \left\lfloor \frac{3}{24} \right\rfloor + \left\lfloor \frac{4}{24} \right\rfloor + \cdots + \left\lfloor \frac{n}{24} \right\rfloor$ is at least 2018?

Note that $\left\lfloor \frac{a}{b} \right\rfloor$ denotes the greatest integer less than or equal to $\frac{a}{b}$.

(A) 322 **(B)** 323 **(C)** 324 **(D)** 325 **(E)** 326

---

17 Find the area of the region enclosed by the graph of $||x| + |y-4|| + ||x| - |y-4|| = 10$ in the xy-coordinate plane.

(A) 25 **(B)** 50 **(C)** 100 **(D)** 200 **(E)** 400

---

18 A permutation of the first seven positive integers is randomly selected. What is the probability that two even integers never appear consecutively in this permutation? An example of such a permutation is $1, 2, 3, 4, 5, 6, 7$ because at no point do two even integers appear next to one another in this list.

(A) $\frac{1}{5}$ **(B)** $\frac{3}{14}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{2}{7}$ **(E)** $\frac{1}{3}$

---

19 Isoceles triangle ABC has side lengths of $AB = AC = 5$ and $BC = 6$. Let D be the foot of the altitude from A to BC, and let lines l and m be the unique lines that pass through D and are perpendicular to AB and AC, respectively. Line l intersects the extension of AC at E and line m intersects the extension of AB at F. Given that the length of EF can be expressed in the form $\frac{m}{n}$, where m and n are relatively prime positive integers, what is $m+n$?

(A) 99 **(B)** 103 **(C)** 107 **(D)** 111 **(E)** 113

---

20 Consider the pairs of positive integers (a, b) with $a > b$ that satisfy the following equation:

$$a^2 + b^2 - 2ab - 2a - 2b = 0$$

Determine the 5th smallest possible value of a.

(A) 24 **(B)** 30 **(C)** 36 **(D)** 42 **(E)** 48

---

21 In square $ABCD, W$ and X lie on AB and AD, respectively, such that $AW = AX = 4$. Similarly, Y and Z lie on BC and CD, respectively, such that $CY = CZ = 2$. Let WZ and XY intersect at I. Given that the area of $CYIZ$ is 5, the area of $ABCD$ can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Define $S(k)$ as the sum of the digits of k. What is $S(m) + S(n)$?

(A) 13 **(B)** 17 **(C)** 21 **(D)** 25 **(E)** 29

---

22 Let N be the coefficient of x^2 in the expansion of $(1+x+x^2)(1+2x+2x^2)(1+3x+3x^2)(1+4x+4x^2)\cdots(1+19x+19x^2)(1+20x+20x^2)$. What is the remainder when N is divided by 1000?

(A) 425 **(B)** 525 **(C)** 625 **(D)** 725 **(E)** 825

---

23 Consider circle O of radius 1. Points A, B, C, D lie on the circle in that order such that arc $AB = BC = DA = 60^\circ$ and arc $CD = 180^\circ$. Let AC intersect BD at I. There exists a circle that is externally tangent to BI and CI and internally tangent to circle O. What is the radius of this circle?

(A) $\frac{1}{5}$ **(B)** $\frac{\sqrt{3}}{8}$ **(C)** $\frac{\sqrt{3}+1}{12}$ **(D)** $\frac{\sqrt{3}-1}{3}$ **(E)** $\frac{1}{4}$

---

24 A permutation a_1, a_2, a_3, a_4, a_5 of the integers $1, 2, 3, 4, 5$ is called *acceptable* if it satisfies either $a_i > a_{i-1}, a_{i+1}$ (a_i is greater than both a_{i-1} and a_{i+1}) or $a_i < a_{i-1}, a_{i+1}$ for $2 \leq i \leq 4$. How many *acceptable* permutations exist?

(A) 24 **(B)** 26 **(C)** 28 **(D)** 30 **(E)** 32

---

25 Pyramid $ABCDE$ has square base $ABCD$ and apex E. One ant is placed at vertex A and another ant is placed at vertex B. Every minute, each ant moves to an adjacent vertex. The expected number of minutes that pass until the ants end up at the same vertex can be expressed in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

(A) 15 **(B)** 38 **(C)** 53 **(D)** 121 **(E)** 445
