National Math Olympiad (Second Round) 2016

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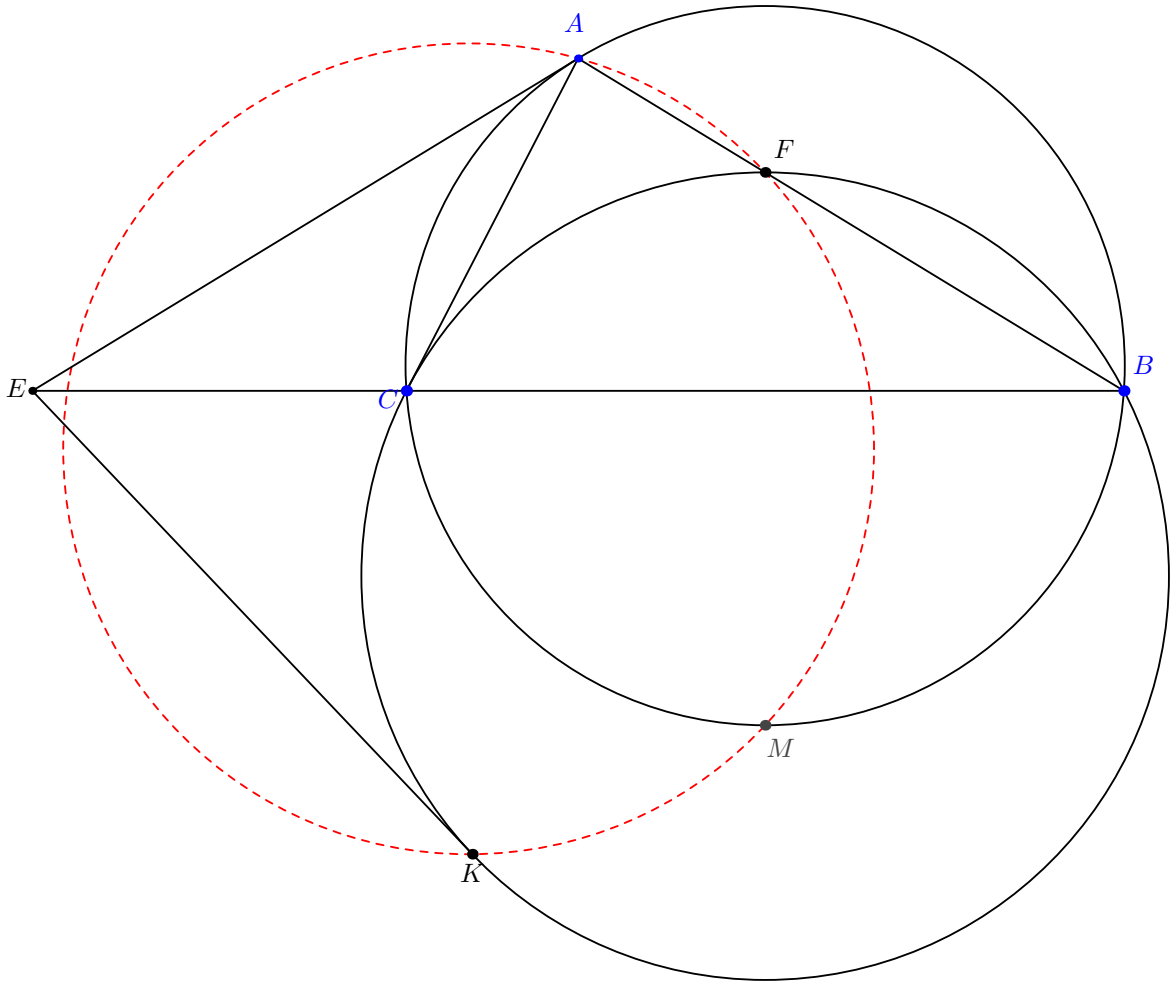
– Day 1

1 If $0 < a \leq b \leq c$ prove that

$$\frac{(c-a)^2}{6c} \leq \frac{a+b+c}{3} - \frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$$

2 Let ABC be a triangle such that $\angle C = 2\angle B$ and ω be its circumcircle. a tangent from A to ω intersect BC at E. Ω is a circle passing throw B that is tangent to AC at C. Let $\Omega \cap AB = F$. K is a point on Ω such that EK is tangent to Ω (A, K aren't in one side of BC). Let M be the midpoint of arc BC of ω (not containing A). Prove that $AFMK$ is a cyclic quadrilateral.



3 A council has 6 members and decisions are based on agreeing and disagreeing votes. We call a decision making method an **Acceptable way to decide** if it satisfies the two following conditions:

Ascending condition: If in some case, the final result is positive, it also stays positive if some one changes their disagreeing vote to agreeing vote.
Symmetry condition: If all members change their votes, the result will also change.

Weighted Voting for example, is an **Acceptable way to decide**. In which members are allotted with non-negative weights like $\omega_1, \omega_2, \cdots, \omega_6$ and the final decision is made with comparing the weight sum of the votes for, and the votes against. For instance if $\omega_1 = 2$ and for all $i \geq 2, \omega_i = 1$, decision is based on the majority of the votes, and in case when votes are equal, the vote of the first member will be the decider.
Give an example of some **Acceptable way to decide** method that cannot be represented as a

	Weighted Voting method.
–	Day 2
4	Let $l_1, l_2, l_3, ..., L_n$ be lines in the plane such that no two of them are parallel and no three of them are concurrent. Let A be the intersection point of lines l_i, l_j. We call A an "Interior Point" if there are points C, D on l_i and E, F on l_j such that A is between C, D and E, F. Prove that there are at least $\frac{(n-2)(n-3)}{2}$ Interior points.($n > 2$) note: by point here we mean the points which are intersection point of two of $l_1, l_2, ..., l_n$.
5	$ABCD$ is a quadrilateral such that $\angle ACB = \angle ACD$. T is inside of $ABCD$ such that $\angle ADC - \angle ATB = \angle BAC$ and $\angle ABC - \angle ATD = \angle CAD$. Prove that $\angle BAT = \angle DAC$.
6	Find all functions $f : \mathbb{N} \to \mathbb{N}$ Such that: 1.for all $x, y \in N$:$x + y \vert f(x) + f(y)$ 2.for all $x \geq 1395$:$x^3 \geq 2f(x)$