Korea National Olympiad 1996

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by PARISsaintGERMAIN

1 If you draw 4 points on the unit circle, prove that you can always find two points where their distance between is less than $\sqrt{2}$.

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2 Let the $f:\mathbb{N}\rightarrow\mathbb{N}$ be the function such that
(i) For all positive integers n, $f(n+f(n))=f(n)$
(ii) $f(n_o)=1$ for some n_0

Prove that $f(n)\equiv 1$.

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3 Let $a=\lfloor \sqrt{n}\rfloor$ for given positive integer n.
Express the summation $\sum_{k=1}^{n}\lfloor \sqrt{k}\rfloor$ in terms of n and a.

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4 Circle C(the center is C.) is inside the $\angle XOY$ and it is tangent to the two sides of the angle. Let C_1 be the circle that passes through the center of C and tangent to two sides of angle and let A be one of the endpoint of diameter of C_1 that passes through C and B be the intersection of this diameter and circle C. Prove that the cirlce that A is the center and AB is the radius is also tangent to the two sides of $\angle XOY$.

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5 Find all integer solution triple (x,y,z) such that $x^2+y^2+z^2-2xyz=0$.

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6 Find the minimum value of k such that there exists two sequence a_i, b_i for $i=1,2,\cdots ,k$ that satisfies the following conditions.

(i) For all $i=1,2,\cdots ,k, a_i,b_i$ is the element of $S=\{1996^n|n=0,1,2,\cdots\}$.
(ii) For all $i=1,2,\cdots ,k, a_i\ne b_i$.
(iii) For all $i=1,2,\cdots ,k, a_i\le a_{i+1}$ and $b_i\le b_{i+1}$.
(iv) $\sum_{i=1}^{k} a_i=\sum_{i=1}^{k} b_i$.

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7 Let A_n be the set of real numbers such that each element of A_n can be expressed as $1+\frac{a_1}{\sqrt{2}}+\frac{a_2}{(\sqrt{2})^2}+\cdots+\frac{a_n}{(\sqrt{n})^n}$ for given n. Find both $|A_n|$ and sum of the products of two distinct elements of A_n where each a_i is either 1 or -1.

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8 Let $\triangle ABC$ be the acute triangle such that $AB\ne AC$. Let V be the intersection of BC and angle bisector of $\angle A$. Let D be the foot of altitude from A to BC. Let E,F be the intersection of circumcircle of $\triangle AVD$ and CA, AB respectively. Prove that the lines AD, BE, CF is concurrent.
