Moroccan Team Selection Test

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– Day 1

1 Let a,b,c be non-negative real numbers such that $a^2+b^2+c^2 \le 3$ then prove that;

$$(a+b+c)(a+b+c-abc) \ge 2(a^2b+b^2c+c^2a)$$

2 The leader of an IMO team chooses positive integers n and k with $n>k$, and announces them to the deputy leader and a contestant. The leader then secretly tells the deputy leader an n-digit binary string, and the deputy leader writes down all n-digit binary strings which differ from the leaders in exactly k positions. (For example, if $n=3$ and $k=1$, and if the leader chooses 101, the deputy leader would write down $001, 111$ and 100.) The contestant is allowed to look at the strings written by the deputy leader and guess the leaders string. What is the minimum number of guesses (in terms of n and k) needed to guarantee the correct answer?

3 Let ABC be a triangle with circumcircle Γ and incenter I and let M be the midpoint of $\overline{BC}$. The points D, E, F are selected on sides $\overline{BC}$, $\overline{CA}$, $\overline{AB}$ such that $\overline{ID} \perp \overline{BC}$, $\overline{IE} \perp \overline{AI}$, and $\overline{IF} \perp \overline{AI}$. Suppose that the circumcircle of $\triangle AEF$ intersects Γ at a point X other than A. Prove that lines XD and AM meet on Γ.

Proposed by Evan Chen, Taiwan

– Day 2

4 Two circles G_1 and G_2 intersect at two points M and N. Let AB be the line tangent to these circles at A and B, respectively, so that M lies closer to AB than N. Let CD be the line parallel to AB and passing through the point M, with C on G_1 and D on G_2. Lines AC and BD meet at E; lines AN and CD meet at P; lines BN and CD meet at Q. Show that $EP=EQ$.

– ii)Prove that EN bisects $\angle CND$.

5 Let n be a positive integer relatively prime to 6. We paint the vertices of a regular n-gon with three colours so that there is an odd number of vertices of each colour. Show that there exists an isosceles triangle whose three vertices are of different colours.

6 For any positive integer k, denote the sum of digits of k in its decimal representation by $S(k)$. Find all polynomials $P(x)$ with integer coefficients such that for any positive integer $n \ge 2016$, the integer $P(n)$ is positive and

$$S(P(n)) = P(S(n)).$$

Proposed by Warut Suksompong, Thailand