Spain Mathematical Olympiad 2018

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–	Day 1
1	Find all positive integers x such that $2x+1$ is a perfect square but none of the integers $2x+2, 2x+3, \ldots, 3x+2$ are perfect squares.
2	Let n be a positive integer. $2n+1$ tokens are in a row, each being black or white. A token is said to be *balanced* if the number of white tokens on its left plus the number of black tokens on its right is n. Determine whether the number of *balanced* tokens is even or odd.
3	Let ABC be an acute-angled triangle with circumcenter O and let M be a point on AB. The circumcircle of AMO intersects AC a second time on K and the circumcircle of BOM intersects BC a second time on N. Prove that $[MNK] \geq \frac{[ABC]}{4}$ and determine the equality case.
–	Day 2
4	Points on a spherical surface with radius 4 are colored in 4 different colors. Prove that there exist two points with the same color such that the distance between them is either $4\sqrt{3}$ or $2\sqrt{6}$. (Distance is Euclidean, that is, the length of the straight segment between the points)
5	Let a, b be coprime positive integers. A positive integer n is said to be *weak* if there do not exist any nonnegative integers x, y such that $ax+by=n$. Prove that if n is a *weak* integer and $n < \frac{ab}{6}$, then there exists an integer $k \geq 2$ such that kn is *weak*.
6	Find all functions such that $f: \mathbb{R}^+ \to \mathbb{R}^+$ and $f(x+f(y)) = yf(xy+1)$ for every $x, y \in \mathbb{R}^+$.