Macedonian National Olympiad 2018

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by steppewolf

Problem 1 Determine all natural numbers n such that $9^n - 7$ can be represented as a product of at least two consecutive natural numbers.

Problem 2 Let n be a natural number and C a non-negative real number. Determine the number of sequences of real numbers $1, x_2, ..., x_n, 1$ such that the absolute value of the difference between any two adjacent terms is equal to C.

Problem 3 Determine all functions $f : \mathbb{R} \to \mathbb{R}$ such that:

$$f(\max\{x, y\} + \min\{f(x), f(y)\}) = x + y$$

for all real $x, y \in \mathbb{R}$

Proposed by Nikola Velov

Problem 4 Let $t_k = a_1^k + a_2^k + ... + a_n^k$, where a_1, a_2, ... a_n are positive real numbers and $k \in \mathbb{N}$. Prove that

$$\frac{t_5^2 t_1^6}{15} - \frac{t_4^4 t_2^2 t_1^2}{6} + \frac{t_2^3 t_4^5}{10} \geq 0$$

Proposed by Daniel Velinov

Problem 5 Given is an acute $\triangle ABC$ with orthocenter H. The point H' is symmetric to H over the side AB. Let N be the intersection point of HH' and AB. The circle passing through A, N and H' intersects AC for the second time in M, and the circle passing through B, N and H' intersects BC for the second time in P. Prove that M, N and P are collinear.

Proposed by Petar Filipovski
