Nepal National Olympiad 2018

www.artofproblemsolving.com/community/c645392

by khan.academy

1a [b]Problem Section#1

a) A set contains four numbers. The six pairwise sums of distinct elements of the set, in no particular order, are $189, 320, 287, 264, x$, and y. Find the greatest possible value of: $x + y$.

NOTE: There is a high chance that this problems was copied.

1b [b]Problem Section#1

b) Let a, b be positive integers such that $b^n + n$ is a multiple of $a^n + n$ for all positive integers n. Prove that $a = b$.

1c [b]Problem Section#1

c) Find all pairs (m, n) of non-negative integers for which $m^2 + 2.3^n = m(2^{n+1} - 1)$.

2a [b]Problem Section#2

a) If

$$ax + by = 7$$
$$ax^2 + by^2 = 49$$
$$ax^3 + by^3 = 133$$
$$ax^4 + by^4 = 406$$

,

find the value of $2014(x + y - xy) - 100(a + b)$.

2b [b]Problem Section#2

b) Find the maximal value of $(x^3 + 1)(y^3 + 1)$, where $x, y \in \mathbb{R}$, $x + y = 1$.

2c [b]Problem Section#2

c). Denote by $\mathbb{Q}^+$ the set of all positive rational numbers. Determine all functions $f : \mathbb{Q}^+ \to \mathbb{Q}^+$ which satisfy the following equation for all $x, y \in \mathbb{Q}^+ : f(f(x)^2.y) = x^3.f(xy)$.

3a [b]Problem Section#3

a) Circles O_1 and O_2 interest at two points B and C, and BC is the diameter of circle O_1. Construct a
tangent line of circle O_1 at C and intersecting circle O_2 at another point A. Join AB to intersect circle O_1 at point E, then join CE and extend it to intersect circle O_2 at point F. Assume H is

an arbitrary
point on line segment AF. Join HE and extend it to intersect circle O_1 at point G, and then join BG
and extend it to intersect the extend of AC at point D. Prove: $\frac{AH}{HF} = \frac{AC}{CD}$.

3b [b] Problem Section#3

NOTE: Neglect that HF and CD.

3c [b]Problem Section#3

c) Let $ABCDE$ be a convex pentagon such that $BC \parallel AE, AB = BC + AE$, and $\angle ABC = \angle CDE$. Let M be the midpoint of CE, and let O be the circumcenter of triangle BCD. Given that $\angle DMO = 90^o$, prove that $2\angle BDA = \angle CDE$.

4a [b]Problem Section#4

a) There is a $6*6$ grid, each square filled with a grasshopper. After the bell rings, each grasshopper
jumps to an adjacent square (A square that shares a side). What is the maximum number of empty
squares possible?

4b [b]Problem Section#4

b) Let A be a unit square. What is the largest area of a triangle whose vertices lie on the perimeter of A? Justify your answer.

4c [b]Problem Section#4

c) A special deck of cards contains 49 cards, each labeled with a number from 1 to 7 and colored with
one of seven colors. Each number-color combination appears on exactly one card. John will select
a set of eight cards from the deck at random. Given that he gets at least one card of each color and
at least one card with each number, the probability that John can discard one of his cards and still
have at least one card of each color and at least one card with each number is $\frac{p}{q}$, where p and q are
relatively prime positive integers. Find $p + q$.
