Finals 2012

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–	Day 1
1	Decide, whether exists positive rational number w, which isn't integer, such that w^w is a rational number.
2	Determine all pairs (m, n) of positive integers, for which cube K with edges of length n, can be build in with cuboids of shape $m \times 1 \times 1$ to create cube with edges of length $n+2$, which has the same center as cube K.
3	Triangle ABC with $AB = AC$ is inscribed in circle o. Circles o_1 and o_2 are internally tangent to circle o in points P and Q, respectively, and they are tangent to segments AB and AC, respectively, and they are disjoint with the interior of triangle ABC. Let m be a line tangent to circles o_1 and o_2, such that points P and Q lie on the opposite side than point A. Line m cuts segments AB and AC in points K and L, respectively. Prove, that intersection point of lines PK and QL lies on bisector of angle BAC.
–	Day 2
4	n players ($n \geq 4$) took part in the tournament. Each player played exactly one match with every other player, there were no draws. There was no four players (A, B, C, D), such that A won with B, B won with C, C won with D and D won with A. Determine, depending on n, maximum number of trios of players (A, B, C), such that A won with B, B won with C and C won with A. (Attention: Trios (A, B, C), (B, C, A) and (C, A, B) are the same trio.)
5	Point O is a center of circumcircle of acute triangle ABC, bisector of angle BAC cuts side BC in point D. Let M be a point such that, $MC \perp BC$ and $MA \perp AD$. Lines BM and OA intersect in point P. Show that circle of center in point P passing through a point A is tangent to line BC.
6	Show that for any positive real numbers a, b, c true is inequality: $\left(\frac{a-b}{c}\right)^2 + \left(\frac{b-c}{a}\right)^2 + \left(\frac{c-a}{b}\right)^2 \geq 2\sqrt{2}\left(\frac{a-b}{c} + \frac{b-c}{a} + \frac{c-a}{b}\right)$.