International Zhautykov Olympiad 2018

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– [b] Day 1

1 Let α, β, γ measures of angles of opposite to the sides of triangle with measures a, b, c respectively.Prove that

$$2(cos^2\alpha + cos^2\beta + cos^2\gamma) \geq \frac{a^2}{b^2+c^2} + \frac{b^2}{a^2+c^2} + \frac{c^2}{a^2+b^2}$$

2 Let N, K, L be points on AB, BC, CA such that CN bisector of angle $\angle ACB$ and $AL = BK$.Let $BL \cap AK = P$.If I, J be incenters of triangles $\triangle BPK$ and $\triangle ALP$ and $IJ \cap CN = Q$ prove that $IQ = JP$

3 Prove that there exist infinitely pairs (m, n) such that $m + n$ divides $(m!)^n + (n!)^m + 1$

– [b] Day 2

4 Crocodile chooses 1 x 4 tile from 2018 x 2018 square.The bear has tilometer that checks 3x3 square of 2018 x 2018 is there any of choosen cells by crocodile.Tilometer says "YES" if there is at least one choosen cell among checked 3 x 3 square.For what is the smallest number of such questions the Bear can certainly get an affirmative answer?

5 Find all real numbers a such that there exist $f : \mathbb{R} \to \mathbb{R}$ with

$$f(x - f(y)) = f(x) + a[y]$$

for all $x, y \in \mathbb{R}$

6 In a circle with a radius R a convex hexagon is inscribed. The diagonals AD and BE,BE and CF,CF and AD of the hexagon intersect at the points M,N and$K$, respectively. Let $r_1, r_2, r_3, r_4, r_5, r_6$ be the radii of circles inscribed in triangles $ABM, BCN, CDK, DEM, EFN, AFK$ respectively. Prove that.

$$r_1 + r_2 + r_3 + r_4 + r_5 + r_6 \leq R\sqrt{3}$$

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