Turkey EGMO TST 2018

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–	[b] Day 1
1	Let $ABCD$ be a cyclic quadrilateral and w be its circumcircle. For a given point E inside w, DE intersects AB at F inside w. Let l be a line passes through E and tangent to circle AEF. Let G be any point on l and inside the quadrilateral $ABCD$. Show that if $\angle GAD = \angle BAE$ and $\angle GCB + \angle GAB = \angle EAD + \angle AGD + \angle ABE$ then BC, AD and EG are concurrent.
2	Determine all pairs (m, n) of positive integers such that $m^2 + n^2 = 2018(m - n)$
3	In how many ways every unit square of a 2018 x 2018 board can be colored in red or white such that number of red unit squares in any two rows are distinct and number of red squares in any two columns are distinct.
–	[b] Day 2
4	There are n stone piles each consisting of 2018 stones. The weight of each stone is equal to one of the numbers $1, 2, 3, ...25$ and the total weights of any two piles are different. It is given that if we choose any two piles and remove the heaviest and lightest stones from each of these piles then the pile which has the heavier one becomes the lighter one. Determine the maximal possible value of n.
5	Prove that $\frac{x^2+1}{(x+y)^2+4(z+1)} + \frac{y^2+1}{(y+z)^2+4(x+1)} + \frac{z^2+1}{(z+x)^2+4(y+1)} \geq \frac{1}{2}$ for all positive reals x, y, z
6	Let $f : \mathbb{Z}_+ \to \mathbb{Z}_+$ is one to one and bijective function. Prove that $f(mn) = f(m)f(n)$ if and only if $lcm(f(m), f(n)) = f(lcm(m, n))$