Balkan MO 2018

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by microsoft_office_word, Hamel

1 A quadrilateral $ABCD$ is inscribed in a circle k where $AB > CD$,and AB is not paralel to CD.Point M is the intersection of diagonals AC and BD, and the perpendicular from M to AB intersects the segment AB at a point E.If EM bisects the angle CED prove that AB is diameter of k.
Proposed by Emil Stoyanov,Bulgaria

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2 Let q be a positive rational number. Two ants are initially at the same point X in the plane. In the n-th minute $(n = 1, 2, ...)$ each of them chooses whether to walk due north, east, south or west and then walks the distance of q^n metres. After a whole number of minutes, they are at the same point in the plane (not necessarily X), but have not taken exactly the same route within that time. Determine all possible values of q.

Proposed by Jeremy King, UK

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3 Alice and Bob play the following game: They start with non-empty piles of coins. Taking turns, with Alice playing first, each player choose a pile with an even number of coins and moves half of the coins of this pile to the other pile. The game ends if a player cannot move, in which case the other player wins.

Determine all pairs (a, b) of positive integers such that if initially the two piles have a and b coins respectively, then Bob has a winning strategy.

Proposed by Dimitris Christophides, Cyprus

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4 Find all primes p and q such that $3p^{q-1} + 1$ divides $11^p + 17^p$

Proposed by Stanislav Dimitrov,Bulgaria
