**Moroccan Team Selection Test 2012**

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– Day 1

**1** Find all prime numbers $p_1, , p_n$ (not necessarily different) such that :

$$\prod_{i=1}^{n} p_i = 10 \sum_{i=1}^{n} p_i$$

**2** Let $(a_n)_{n\geq 1}$ be an increasing sequence of positive integers such that $a_1 = 1$, and for all positive integers $n$, $a_{n+1} \leq 2n$.
Prove that for every positive $n$; there exists positive integers $p$ and $q$ such that $n = a_p - a_q$.

**3** $a_1, , a_n$ are real numbers such that $a_1 + + a_n = 0$ and $|a_1| + + |a_n| = 1$. Prove that :

$$|a_1 + 2a_2 + + na_n| \leq \frac{n-1}{2}$$

**4** $ABC$ is a non-isosceles triangle. $O, I, H$ are respectively the center of its circumscribed circle, the inscribed circle and its orthocenter.
prove that $\widehat{OIH}$ is obtuse.

– Day 2

**1** Find all positive integers $n, k$ such that $(n-1)! = n^k - 1$.

**2** Find all positive integer $n$ and prime number $p$ such that $p^2 + 7^n$ is a perfect square

**3** Find the maximal value of the following expression, if $a, b, c$ are nonnegative and $a + b + c = 1$.

$$\frac{1}{a^2 - 4a + 9} + \frac{1}{b^2 - 4b + 9} + \frac{1}{c^2 - 4c + 9}$$

**4** Let $ABC$ be an acute triangle with circumcircle $\Omega$. Let $B_0$ be the midpoint of $AC$ and let $C_0$ be the midpoint of $AB$. Let $D$ be the foot of the altitude from $A$ and let $G$ be the centroid of the triangle $ABC$. Let $\omega$ be a circle through $B_0$ and $C_0$ that is tangent to the circle $\Omega$ at a point $X \neq A$. Prove that the points $D, G$ and $X$ are collinear.

*Proposed by Ismail Isaev and Mikhail Isaev, Russia*