Azerbaijan Team Selection Test 2016
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Day 1 May 19th

1 Tangents from the point A to the circle Γ touche this circle at C and D.Let B be a point on Γ,different from C and D. The circle ω that passes through points A and B intersect with lines AC and AD at F and E,respectively.Prove that the circumcircles of triangles ABC and DEB are tangent if and only if the points C, D, F and E are cyclic.

2 Find all polynomials $P(x)$ with real coefficents, such that for all x, y, z satisfying $x+y+z=0$, the equation below is true:

$$P(x+y)^3+P(y+z)^3+P(z+x)^3=3P((x+y)(y+z)(z+x))$$

Day 2 May 20th

1 Determine all positive integers M such that the sequence $a_0, a_1, a_2, \cdots$ defined by

$$a_0=M+\frac{1}{2} \qquad \text{and} \qquad a_{k+1}=a_k\lfloor a_k\rfloor \quad \text{for } k=0,1,2,\cdots$$

contains at least one integer term.

2 Let ABC be a triangle with $\angle C=90^\circ$, and let H be the foot of the altitude from C. A point D is chosen inside the triangle CBH so that CH bisects AD. Let P be the intersection point of the lines BD and CH. Let ω be the semicircle with diameter BD that meets the segment CB at an interior point. A line through P is tangent to ω at Q. Prove that the lines CQ and AD meet on ω.

3 Prove that there does not exist a function $f:\mathbb{R}^+\to\mathbb{R}^+$ such that

$$f(f(x)+y)=f(x)+3x+yf(y)$$

for all positive reals x, y.

Day 3 May 21st

1 Suppose that a sequence $a_1, a_2, \ldots$ of positive real numbers satisfies

$$a_{k+1}\geq\frac{ka_k}{a_k^2+(k-1)}$$

for every positive integer k. Prove that $a_1 + a_2 + \ldots + a_n \geq n$ for every $n \geq 2$.

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2 A positive interger n is called *rising* if its decimal representation $a_k a_{k-1} \cdots a_0$ satisfies the condition $a_k \leq a_{k-1} \leq \cdots \leq a_0$. Polynomial P with real coefficents is called *interger-valued* if for all integer numbers n, $P(n)$ takes interger values. $P(n)$ is called *rising-valued* if for all *rising* numbers n, $P(n)$ takes integer values.
Does it necessarily mean that, "every *rising-valued* P is also *interger-valued* P"?

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3 During a day 2016 customers visited the store. Every customer has been only once at the store(a customer enters the store,spends some time, and leaves the store). Find the greatest integer k that makes the following statement always true.

We can find k customers such that either all of them have been at the store at the same time, or any two of them have not been at the same store at the same time.
