Regional Competition For Advanced Students 2017, held on April 30, 2017

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1 Let $x_1, x_2, \ldots, x_n$ be non-negative real numbers such that

$$x_1^2 + x_2^2 + \ldots x_9^2 \geq 25.$$

Prove that one can choose three of these numbers such that their sum is at least 5.

Proposed by Karl Czakler

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2 Let $ABCD$ be a cyclic quadrilateral with perpendicular diagonals and circumcenter O. Let g be the line obtained by reflection of the diagonal AC along the angle bisector of $\angle BAD$. Prove that the point O lies on the line g.

Proposed by Theresia Eisenklbl

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3 The nonnegative integers 2000, 17 and n are written on the blackboard. Alice and Bob play the following game: Alice begins, then they play in turns. A move consists in replacing one of the three numbers by the absolute difference of the other two. No moves are allowed, where all three numbers remain unchanged. The player who is in turn and cannot make an allowed move loses the game.

- Prove that the game will end for every number n.
- Who wins the game in the case $n = 2017$?

Proposed by Richard Henner

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4 Determine all integers $n \geq 2$, satisfying

$$n = a^2 + b^2,$$

where a is the smallest divisor of n different from 1 and b is an arbitrary divisor of n.
Proposed by Walther Janous
