2018 Cyprus IMO TST Problems

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1 Determine all integers $n \geq 2$ for which the number 11111 in base n is a perfect square.

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2 Consider a trapezium $AB\Gamma\Delta$, where $A\Delta \parallel B\Gamma$ and $\measuredangle A = 120^\circ$. Let E be the midpoint of AB and let O_1 and O_2 be the circumcenters of triangles $AE\Delta$ and $BE\Gamma$, respectively. Prove that the area of the trapezium is equal to six time the area of the triangle O_1EO_2.

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3 Find all triples (α, β, γ) of positive real numbers for which the expression

$$K = \frac{\alpha + 3\gamma}{\alpha + 2\beta + \gamma} + \frac{4\beta}{\alpha + \beta + 2\gamma} - \frac{8\gamma}{\alpha + \beta + 3\gamma}$$

obtains its minimum value.

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4 Let $\Lambda = \{1, 2, \dots, 2v-1, 2v\}$ and $P = \{\alpha_1, \alpha_2, \dots, \alpha_{2v-1}, \alpha_{2v}\}$ be a permutation of the elements of Λ.

(a) Prove that

$$\sum_{i=1}^{v} \alpha_{2i-1}\alpha_{2i} \leq \sum_{i=1}^{v} (2i-1)2i.$$

(b) Determine the largest positive integer m such that we can partition the $m \times m$ square into 7 rectangles for which every pair of them has no common interior points and their lengths and widths form the following sequence:

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14.$$

Source Taken from Cyprus national math olympiad webpage (`http://www.cms.org.cy/index.php?id=1096`).
