**Spain Mathematical Olympiad 2005**

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| – | Day 1 |
| **1** | Let $a$ and $b$ be integers. Demonstrate that the equation $$(x-a)(x-b)(x-3)+1=0$$ has an integer solution. |
| **2** | Is it possible to color points in the Cartesian Plane $(x, y)$ with integer coordinates with three colors, such that each color appears infinitely many times on infinitely many lines parallel to the $x$-axis and that any three points, each of a different color, are not in a line? Justify your answer. |
| **3** | We will say that a triangle is multiplicative if the product of the heights of two of its sides is equal to the length of the third side. Given $ABC\ldots XYZ$ is a regular polygon with $n$ sides of length $1$. The $n-3$ diagonals that go out from vertex $A$ divide the triangle $ZAB$ in $n-2$ smaller triangles. Prove that each one of these triangles is multiplicative. |
| – | Day 2 |
| **1** | Prove that for every positive integer $n$, the decimal expression of $\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}$ is periodic . |
| **2** | Let $r, s, u, v$ be real numbers. Prove that: $$min\{r-s^2, s-u^2, u-v^2, v-r^2\} \leq \frac{1}{4}$$ |
| **3** | In a triangle with sides $a, b, c$ the side $a$ is the arithmetic mean of $b$ and $c$. Prove that:<br>a) $0^o \leq A \leq 60^o$.<br>b) The height relative to side $a$ is three times the inradius $r$.<br>c) The distance from the circumcenter to side $a$ is $R-r$, where $R$ is the circumradius. |