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by parmenides51

**1**    Find all real $x$ such that $\sqrt{x - \frac{1}{x}} + \sqrt{1 - \frac{1}{x}} > \frac{x-1}{x}$

**2**    Show that there are $1977$ non-similar triangles such that the angles $A, B, C$ satisfy $\frac{\sin A + \sin B + \sin C}{\cos A + \cos B + \cos C} = \frac{12}{7}$ and $\sin A \sin B \sin C = \frac{12}{25}$.

**3**    Into how many regions do $n$ circles divide the plane, if each pair of circles intersects in two points and no point lies on three circles?

**4**    $p(x)$ is a real polynomial of degree $3$.
Find necessary and sufficient conditions on its coefficients in order that $p(n)$ is integral for every integer $n$.

**5**    The real numbers $a_0, a_1, ..., a_{n+1}$ satisfy $a_0 = a_{n+1} = 0$ and $|a_{k-1} - 2a_k + a_{k+1}| \le 1$ for $k = 1, 2, ..., n$. Show that $|a_k| \le \frac{k(n+1-k)}{2}$ for all $k$.

**6**    The planes $p$ and $p'$ are parallel. A polygon $P$ on $p$ has $m$ sides and a polygon $P'$ on $p'$ has $n$ sides. Find the largest and smallest distances between a vertex of $P$ and a vertex of $P'$.