Mexico National Olympiad 2006

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–	Day 1
1	Let ab be a two digit number. A positive integer n is a *relative* of ab if: - The units digit of n is b. - The remaining digits of n are nonzero and add up to a. Find all two digit numbers which divide all of their relatives.
2	Let ABC be a right triangle with a right angle at A, such that $AB < AC$. Let M be the midpoint of BC and D the intersection of AC with the perpendicular on BC passing through M. Let E be the intersection of the parallel to AC that passes through M, with the perpendicular on BD passing through B. Show that the triangles AEM and MCA are similar if and only if $\angle ABC = 60^o$.
3	Let n be an integer greater than 1. In how many ways can we fill all the numbers $1, 2, ..., 2n$ in the boxes of a grid of $2 \times n$, one in each box, so that any two consecutive numbers are they in squares that share one side of the grid?
–	Day 2
4	For which positive integers n can be covered a ladder like that of the figure (but with n steps instead of 4) with n squares of integer sides, not necessarily the same size, without these squares overlapping and without standing out from the edge of the figure ?
5	Let ABC be an acute triangle , with altitudes AD, BE and CF. Circle of diameter AD intersects the sides AB, AC in M, N respevtively. Let P, Q be the intersection points of AD with EF and MN respectively. Show that Q is the midpoint of PD.
6	Let n be the sum of the digits in a natural number A. The number A it's said to be "surtido" if every number 1,2,3,4....,n can be expressed as a sum of digits in A. a)Prove that, if 1,2,3,4,5,6,7,8 are sums of digits in A, then A is "Surtido" b)If 1,2,3,4,5,6,7 are sums of digits in A, does it follow that A is "Surtido"?