Mexico National Olympiad 2004

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–	Day 1
1	Find all the prime number p, q and r with $p < q < r$, such that $25pq + r = 2004$ and $pqr + 1$ is a perfect square.
2	Find the maximum number of positive integers such that any two of them a, b (with $a \neq b$) satisfy that$\|a - b\| \geq \frac{ab}{100}$.
3	Let Z and Y be the tangency points of the incircle of the triangle ABC with the sides AB and CA, respectively. The parallel line to YZ through the midpoint M of BC, meets CA in N. Let L be the point in CA such that $NL = AB$ (and L on the same side of N than A). The line ML meets AB in K. Prove that $KA = NC$.
–	Day 2
4	At the end of a soccer tournament in which any pair of teams played between them exactly once, and in which there were not draws, it was observed that for any three teams A, B and C, if A defeated B and B defeated C, then A defeated C. Any team calculated the difference (positive) between the number of games that it won and the number of games it lost. The sum of all these differences was 5000. How many teams played in the tournament? Find all possible answers.
5	Let ω_1 and ω_2 be two circles such that the center O of ω_2 lies in ω_1. Let C and D be the two intersection points of the circles. Let A be a point on ω_1 and let B be a point on ω_2 such that AC is tangent to ω_2 in C and BC is tangent to ω_1 in C. The line segment AB meets ω_2 again in E and also meets ω_1 again in F. The line CE meets ω_1 again in G and the line CF meets the line GD in H. Prove that the intersection point of GO and EH is the center of the circumcircle of the triangle DEF.
6	What is the maximum number of possible change of directions in a path traveling on the edges of a rectangular array of 2004×2004, if the path does not cross the same place twice?.