**Mexico National Olympiad 2003**

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| – | Day 1 |
| **1** | Find all positive integers with two or more digits such that if we insert a $0$ between the units and tens digits we get a multiple of the original number. |
| **2** | $A, B, C$ are collinear with $B$ betweeen $A$ and $C$. $K_1$ is the circle with diameter $AB$, and $K_2$ is the circle with diameter $BC$. Another circle touches $AC$ at $B$ and meets $K_1$ again at $P$ and $K_2$ again at $Q$. The line $PQ$ meets $K_1$ again at $R$ and $K_2$ again at $S$. Show that the lines $AR$ and $CS$ meet on the perpendicular to $AC$ at $B$. |
| **3** | At a party there are $n$ women and $n$ men. Each woman likes $r$ of the men, and each man likes $s$ of then women. For which $r$ and $s$ must there be a man and a woman who like each other? |
| – | Day 2 |
| **4** | The quadrilateral $ABCD$ has $AB$ parallel to $CD$. $P$ is on the side $AB$ and $Q$ on the side $CD$ such that $\frac{AP}{PB} = \frac{DQ}{CQ}$. M is the intersection of $AQ$ and $DP$, and $N$ is the intersection of $PC$ and $QB$. Find $MN$ in terms of $AB$ and $CD$. |
| **5** | Some cards each have a pair of numbers written on them. There is just one card for each pair $(a, b)$ with $1 \leq a < b \leq 2003$. Two players play the following game. Each removes a card in turn and writes the product $ab$ of its numbers on the blackboard. The first player who causes the greatest common divisor of the numbers on the blackboard to fall to $1$ loses. Which player has a winning strategy? |
| **6** | Given a positive integer $n$, an allowed move is to form $2n+1$ or $3n+2$. The set $S_n$ is the set of all numbers that can be obtained by a sequence of allowed moves starting with $n$. For example, we can form $5 \to 11 \to 35$ so $5, 11$ and $35$ belong to $S_5$. We call $m$ and $n$ compatible if $S_m$ and $S_n$ has a common element. Which members of $\{1, 2, 3, ..., 2002\}$ are compatible with $2003$? |