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by parmenides51

- 1 Nine journalists from different countries attend a press conference. None of these speaks more than three languages, and each pair of the journalists share a common language. Show that there are at least five journalists sharing a common language.

- 2 Let $ABCD$ be a parallelogram in the plane. We draw two circles of radius R , one through the points A and B , the other through B and C . Let E be the other intersection point of the circles. We assume that E is not a vertex of the parallelogram. Show that the circle passing through A , D , and E also has radius R .

- 3 Let f be a strictly increasing function defined in the set of natural numbers satisfying the conditions $f(2) = a > 2$ and $f(mn) = f(m)f(n)$ for all natural numbers m and n . Determine the smallest possible value of a .

- 4 Let a, b , and c be positive real numbers. Prove: $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \leq \frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{a^2}$.
