



Nordic 1988

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- 1 The positive integer n has the following property:
if the three last digits of n are removed, the number $\sqrt[3]{n}$ remains.
Find n .

- 2 Let a, b , and c be non-zero real numbers and let $a \geq b \geq c$. Prove the inequality $\frac{a^3-c^3}{3} \geq abc\left(\frac{a-b}{c} + \frac{b-c}{a}\right)$. When does equality hold?

- 3 Two concentric spheres have radii r and $R, r < R$. We try to select points A, B and C on the surface of the larger sphere such that all sides of the triangle ABC would be tangent to the surface of the smaller sphere. Show that the points can be selected if and only if $R \leq 2r$.

- 4 Let m_n be the smallest value of the function $f_n(x) = \sum_{k=0}^{2n} x^k$
Show that $m_n \rightarrow \frac{1}{2}$, as $n \rightarrow \infty$.

