Nordic 1989

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1 Find a polynomial P of lowest possible degree such that
(a) P has integer coefficients,
(b) all roots of P are integers,
(c) $P(0) = -1$,
(d) $P(3) = 128$.

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2 Three sides of a tetrahedron are right-angled triangles having the right angle at their common vertex. The areas of these sides are A, B, and C. Find the total surface area of the tetrahedron.

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3 Let S be the set of all points t in the closed interval $[-1, 1]$ such that for the sequence $x_0, x_1, x_2, ...$ defined by the equations $x_0 = t, x_{n+1} = 2x_n^2 - 1$, there exists a positive integer N such that $x_n = 1$ for all $n \geq N$. Show that the set S has infinitely many elements.

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4 For which positive integers n is the following statement true:
if $a_1, a_2, ..., a_n$ are positive integers, $a_k \leq n$ for all k and $\sum\limits_{k=1}^{n} a_k = 2n$
then it is always possible to choose $a_{i1}, a_{i2}, ..., a_{ij}$ in such a way that
the indices $i_1, i_2, ..., i_j$ are different numbers, and $\sum\limits_{k=1}^{j} a_{ik} = n$?
