Nordic 1990

www.artofproblemsolving.com/community/c691090

by parmenides51

---

1 Let $m, n,$ and p be odd positive integers. Prove that the number $\sum_{k=1}^{(n-1)^p} k^m$ is divisible by n

---

2 Let $a_1, a_2, ..., a_n$ be real numbers. Prove $\sqrt[3]{a_1^3 + a_2^3 + ... + a_n^3} \le \sqrt{a_1^2 + a_2^2 + ... + a_n^2}$ (1)
When does equality hold in (1)?

---

3 Let ABC be a triangle and let P be an interior point of ABC. We assume that a line l, which passes through P, but not through A, intersects AB and AC (or their extensions over B or C) at Q and R, respectively. Find l such that the perimeter of the triangle AQR is as small as possible.

---

4 It is possible to perform three operations $f, g,$ and h for positive integers: $f(n) = 10n, g(n) = 10n + 4$, and $h(2n) = n$; in other words, one may write 0 or 4 in the end of the number and one may divide an even number by 2. Prove: every positive integer can be constructed starting from 4 and performing a finite number of the operations $f, g,$ and h in some order.
