Nordic 1991

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1	Determine the last two digits of the number $2^5 + 2^{5^2} + 2^{5^3} + ... + 2^{5^{1991}}$, written in decimal notation.
2	In the trapezium $ABCD$ the sides AB and CD are parallel, and E is a fixed point on the side AB. Determine the point F on the side CD so that the area of the intersection of the triangles ABF and CDE is as large as possible.
3	Show that $\frac{1}{2^2} + \frac{1}{3^2} + ... + \frac{1}{n^2} < \frac{2}{3}$ for all $n \geq 2$.
4	Let $f(x)$ be a polynomial with integer coefficients. We assume that there exists a positive integer k and k consecutive integers $n, n+1, ..., n+k-1$ so that none of the numbers $f(n), f(n+1), ..., f(n+k-1)$ is divisible by k. Show that the zeroes of $f(x)$ are not integers.