Nordic 1992

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1 Determine all real numbers $x > 1, y > 1$, and $z > 1$,satisfying the equation $x + y + z + \frac{3}{x-1} + \frac{3}{y-1} + \frac{3}{z-1} = 2(\sqrt{x+2} + \sqrt{y+2} + \sqrt{z+2})$

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2 Let $n > 1$ be an integer and let $a_1, a_2, ..., a_n$ be n different integers. Show that the polynomial $f(x) = (x-a_1)(x-a_2)\cdot ... \cdot (x-a_n)-1$ is not divisible by any polynomial with integer coefficients and of degree greater than zero but less than n and such that the highest power of x has coefficient 1.

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3 Prove that among all triangles with inradius 1, the equilateral one has the smallest perimeter .

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4 Peter has many squares of equal side. Some of the squares are black, some are white. Peter wants to assemble
a big square, with side equal to n sides of the small squares, so that the big square has no rectangle formed by the small squares such that all the squares in the vertices of the rectangle are of equal colour. How big a square is Peter able to assemble?
