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1 Let O be an interior point in the equilateral triangle ABC, of side length a. The lines AO, BO, and CO intersect the sides of the triangle in the points A_1, B_1, and C_1. Show that $OA_1+OB_1+OC_1 < a$.

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2 We call a finite plane set S consisting of points with integer coefficients a two-neighbour set, if for each point (p,q) of S exactly two of the points $(p+1,q),(p,q+1),(p-1,q),(p,q-1)$ belong to S. For which integers n there exists a two-neighbour set which contains exactly n points?

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3 A piece of paper is the square $ABCD$. We fold it by placing the vertex D on the point D' of the side BC. We assume that AD moves on the segment $A'D'$ and that $A'D'$ intersects AB at E. Prove that the perimeter of the triangle EBD' is one half of the perimeter of the square.

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4 Determine all positive integers $n < 200$, such that $n^2+(n+1)^2$ is the square of an integer.
