Nordic 1995

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1 Let AB be a diameter of a circle with centre O. We choose a point C on the circumference of the circle such that OC and AB are perpendicular to each other. Let P be an arbitrary point on the (smaller) arc BC and let the lines CP and AB meet at Q. We choose R on AP so that RQ and AB are perpendicular to each other. Show that $BQ = QR$.

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2 Messages are coded using sequences consisting of zeroes and ones only. Only sequences with at most two consecutive ones or zeroes are allowed. (For instance the sequence 011001 is allowed, but 011101 is not.) Determine the number of sequences consisting of exactly 12 numbers.

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3 Let $n \geq 2$ and let $x_1, x_2, ..., x_n$ be real numbers satisfying $x_1 + x_2 + ... + x_n \geq 0$ and $x_1^2 + x_2^2 + ... + x_n^2 = 1$. Let $M = max\{x_1, x_2, ..., x_n\}$. Show that $M \geq \frac{1}{\sqrt{n(n-1)}}$ (1) .When does equality hold in (1)?

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4 Show that there exist infinitely many mutually non- congruent triangles T, satisfying
(i) The side lengths of T are consecutive integers.
(ii) The area of T is an integer.
