**Cono Sur Olympiad 2009**

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– Day 1

1 The four circles in the figure determine 10 bounded regions. $10$ numbers summing to $100$ are written in these regions, one in each region. The sum of the numbers contained in each circle is equal to $S$ (the same quantity for each of the four circles). Determine the greatest and smallest possible values of $S$.

2 A *hook* consists of three segments of longitude $1$ forming two right angles as demonstrated in the figure.
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png

We have a square of side length $n$ divided into $n^2$ squares of side length $1$ by lines parallel to its sides. Hooks are placed on this square in such a way that each segment of the hook covers one side of a little square. Two segements of a hook cannot overlap.
Determine all possible values of $n$ for which it is possible to cover the sides of the $n^2$ small squares.

3 Let $A$, $B$, and $C$ be three points such that $B$ is the midpoint of segment $AC$ and let $P$ be a point such that $< PBC = 60$. Equilateral triangle $PCQ$ is constructed such that $B$ and $Q$ are on different half=planes with respect to $PC$, and the equilateral triangle $APR$ is constructed in such a way that $B$ and $R$ are in the same half-plane with respect to $AP$. Let $X$ be the point of intersection of the lines $BQ$ and $PC$, and let $Y$ be the point of intersection of the lines $BR$ and $AP$. Prove that $XY$ and $AC$ are parallel.

– Day 2

4 Andrea and Bruno play a game on a table with $11$ rows and $9$ columns. First Andrea divides the table in $33$ zones. Each zone is formed by $3$ contiguous cells aligned vertically or horizontally, as shown in the figure.
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png
Then, Bruno writes one of the numbers $0, 1, 2, 3, 4, 5$ in each cell in such a way that the sum of the numbers in each zone is equal to $5$. Bruno wins if the sum of the numbers written in each of the $9$ columns of the table is a prime number. Otherwise, Andrea wins. Show that Bruno always has a winning strategy.

**5** Given a succession $C$ of $1001$ positive real numbers (not necessarily distinct), and given a set $K$ of distinct positive integers, the permitted operation is: select a number $k \in K$, then select $k$ numbers in $C$, calculate the arithmetic mean of those $k$ numbers, and replace each of those $k$ selected numbers with the mean.

If $K$ is a set such that for each $C$ we can reach, by a sequence of permitted operations, a state where all the numbers are equal, determine the smallest possible value of the maximum element of $K$.

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**6** Sebastian has a certain number of rectangles with areas that sum up to 3 and with side lengths all less than or equal to $1$. Demonstrate that with each of these rectangles it is possible to cover a square with side $1$ in such a way that the sides of the rectangles are parallel to the sides of the square.

**Note:** The rectangles can overlap and they can protrude over the sides of the square.

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