AoPS Online
Art of Problem Solving

Cono Sur Olympiad 2008

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– Day 1

1 We define $I(n)$ as the result when the digits of n are reversed. For example, $I(123) = 321$, $I(2008) = 8002$. Find all integers n, $1 \leq n \leq 10000$ for which $I(n) = \lceil \frac{n}{2} \rceil$.
Note: $\lceil x \rceil$ denotes the smallest integer greater than or equal to x. For example, $\lceil 2.1 \rceil = 3$, $\lceil 3.9 \rceil = 4$, $\lceil 7 \rceil = 7$.

2 Let P be a point in the interior of triangle ABC. Let X, Y, and Z be points on sides BC, AC, and AB respectively, such that $< PXC =< PYA =< PZB$.
Let U, V, and W be points on sides BC, AC, and AB, respectively, or on their extensions if necessary, with X in between B and U, Y in between C and V, and Z in between A and W, such that $PU = 2PX$, $PV = 2PY$, and $PW = 2PZ$. If the area of triangle XYZ is 1, find the area of triangle UVW.

3 Two friends A and B must solve the following puzzle. Each of them receives a number from the set $\{1, 2, , 250\}$, but they dont see the number that the other received. The objective of each friend is to discover the other friends number. The procedure is as follows: each friend, by turns, announces various not necessarily distinct positive integers: first A says a number, then B says one, A says a number again, etc., in such a way that the sum of all the numbers said is 20. Demonstrate that there exists a strategy that A and B have previously agreed on such that they can reach the objective, no matter which number each one received at the beginning of the puzzle.

– Day 2

4 What is the largest number of cells that can be colored in a 7×7 table in such a way that any 2×2 subtable has at most 2 colored cells?

5 Let ABC be an isosceles triangle with base AB. A semicircle Γ is constructed with its center on the segment AB and which is tangent to the two legs, AC and BC. Consider a line tangent to Γ which cuts the segments AC and BC at D and E, respectively. The line perpendicular to AC at D and the line perpendicular to BC at E intersect each other at P. Let Q be the foot of the perpendicular from P to AB. Show that
$\frac{PQ}{CP} = \frac{1}{2}\frac{AB}{AC}$.

6 A palindrome is a number that is the same when its digits are reversed. Find all numbers that have at least one multiple that is a palindrome.