Cono Sur Olympiad 2004

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by parmenides51, fprosk

–	Day 1
1	Maxi chose 3 digits, and by writing down all possible permutations of these digits, he obtained 6 distinct 3-digit numbers. If exactly one of those numbers is a perfect square and exactly three of them are prime, find Maxis 3 digits. Give all of the possibilities for the 3 digits.
2	Given a circle C and a point P on its exterior, two tangents to the circle are drawn through P, with A and B being the points of tangency. We take a point Q on the minor arc AB of C. Let M be the intersection of AQ with the line perpendicular to AQ that goes through P, and let N be the intersection of BQ with the line perpendicular to BQ that goes through P. Show that, by varying Q on the minor arc AB, all of the lines MN pass through the same point.
3	Let n be a positive integer. We call C_n the number of positive integers x less than 10^n such that the sum of the digits of $2x$ is less than the sum of the digits of x. Show that $C_n \geq \frac{4}{9}(10^n - 1)$.
–	Day 2
4	Arnaldo selects a nonnegative integer a and Bernaldo selects a nonnegative integer b. Both of them secretly tell their number to Cernaldo, who writes the numbers 5, 8, and 15 on the board, one of them being the sum $a + b$. Cernaldo rings a bell and Arnaldo and Bernaldo, individually, write on different slips of paper whether they know or not which of the numbers on the board is the sum $a + b$ and they turn them in to Cernaldo. If both of the papers say NO, Cernaldo rings the bell again and the process is repeated. It is known that both Arnaldo and Bernaldo are honest and intelligent. What is the maximum number of times that the bell can be rung until one of them knows the sum? Personal note: They really phoned it in with the names there
5	Using cardboard equilateral triangles of side length 1, an equilateral triangle of side length 2^{2004} is formed. An equilateral triangle of side 1 whose center coincides with the center of the large triangle is removed. Determine if it is possible to completely cover the remaining surface, without overlaps or holes,

using only pieces in the shape of an isosceles trapezoid, each of which is created by joining three equilateral triangles of side 1.

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6 Let m, n be positive integers. On an $m \times n$ checkerboard, divided into 1×1 squares, we consider all paths that go from upper right vertex to the lower left vertex, travelling exclusively on the grid lines by going down or to the left. We define the area of a path as the number of squares on the checkerboard that are below this path. Let p be a prime such that $r_p(m) + r_p(n) \geq p$, where $r_p(m)$ denotes the remainder when m is divided by p and $r_p(n)$ denotes the remainder when n is divided by p.
How many paths have an area that is a multiple of p?
