Cono Sur Olympiad 2003

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– Day 1

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1 In a soccer tournament between four teams, A, B, C, and D, each team plays each of the others exactly once.
a) Decide if, at the end of the tournament, it is possible for the quantities of goals scored and goals allowed for each team to be as follows:

	A	B	C	D
Goals scored	1	3	6	7
Goals allowed	4	4	4	5

If the answer is yes, give an example for the results of the six games; in the contrary, justify your answer.
b) Decide if, at the end of the tournament, it is possible for the quantities of goals scored and goals allowed for each team to be as follows:

	A	B	C	D
Goals scored	1	3	6	13
Goals allowed	4	4	4	11

If the answer is yes, give an example for the results of the six games; in the contrary, justify your answer.

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2 Define the sequence $\{a_n\}$ in the following manner. $a_1 = 1$ $a_2 = 3$ $a_{n+2} = 2a_{n+1}a_n + 1$; for all $n \geq 1$
Prove that the largest power of 2 that divides $a_{4006} - a_{4005}$ is 2^{2003}.

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3 Let ABC be an acute triangle such that $\angle B = 60$. The circle with diameter AC intersects the internal angle bisectors of A and C at the points M and N, respectively $(M \neq A, N \neq C)$. The internal bisector of $\angle B$ intersects MN and AC at the points R and S, respectively. Prove that $BR \leq RS$.

– Day 2

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4 In an acute triangle ABC, the points H, G, and M are located on BC in such a way that AH, AG, and AM are the height, angle bisector, and median of the triangle, respectively. It is known that $HG = GM$, $AB = 10$, and $AC = 14$. Find the area of triangle ABC.

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5 Let $n = 3k + 1$, where k is a positive integer. A triangular arrangement of side n is formed using circles with the same radius, as is shown in the figure for $n = 7$.
Determine, for each k, the largest number of circles that can be colored red in such a way that there are no two mutually tangent circles that are both colored red.

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6 Show that there exists a sequence of positive integers $x_1, x_2, x_n,$ that satisfies the following two conditions:
(i) Every positive integer appears exactly once,
(ii) For every $n = 1, 2,$ the partial sum $x_1 + x_2 + \quad + x_n$ is divisible by n^n.