

Junior Balkan MO 2018

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1 Find all integers m and n such that the fifth power of m minus the fifth power of n is equal to $16mn$.

2 Find max number n of numbers of three digits such that :

1. Each has digit sum 9
2. No one contains digit 0
3. Each 2 have different unit digits
4. Each 2 have different decimal digits
5. Each 2 have different hundreds digits

3 Let $k > 1$ be a positive integer and $n > 2018$ an odd positive integer. The non-zero rational numbers x_1, x_2, \dots, x_n are not all equal and:

$$x_1 + \frac{k}{x_2} = x_2 + \frac{k}{x_3} = x_3 + \frac{k}{x_4} = \dots = x_{n-1} + \frac{k}{x_n} = x_n + \frac{k}{x_1}$$

Find the minimum value of k , such that the above relations hold.

4 Let $\triangle ABC$ and A', B', C' the symmetric of vertex over opposite sides. The intersection of the circumcircles of $\triangle ABB'$ and $\triangle ACC'$ is A_1, B_1 and C_1 are defined similarly. Prove that lines AA_1, BB_1 and CC_1 are concurrent.
