

Brazil National Olympiad 1980

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- 1 Box A contains black balls and box B contains white balls. Take a certain number of balls from A and place them in B . Then take the same number of balls from B and place them in A . Is the number of white balls in A then greater, equal to, or less than the number of black balls in B ?

- 2 Show that for any positive integer $n > 2$ we can find n distinct positive integers such that the sum of their reciprocals is 1.

- 3 Given a triangle ABC and a point P_0 on the side AB . Construct points P_i, Q_i, R_i as follows. Q_i is the foot of the perpendicular from P_i to BC , R_i is the foot of the perpendicular from Q_i to AC and P_i is the foot of the perpendicular from R_{i-1} to AB . Show that the points P_i converge to a point P on AB and show how to construct P .

- 4 Given 5 points of a sphere radius r , show that two of the points are a distance $\leq r\sqrt{2}$ apart.
