Brazil National Olympiad 1982

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1 The angles of the triangle ABC satisfy $\angle A/\angle C = \angle B/\angle A = 2$. The incenter is $O.K, L$ are the excenters of the excircles opposite B and A respectively. Show that triangles ABC and OKL are similar.

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2 Any positive integer n can be written in the form $n = 2^b(2c+1)$. We call $2c+1$ the *odd part* of n. Given an odd integer $n > 0$, define the sequence $a_0, a_1, a_2, ...$ as follows: $a_0 = 2^n - 1$, a_{k+1} is the *odd part* of $3a_k + 1$. Find a_n.

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3 S is a $(k+1) \times (k+1)$ array of lattice points. How many squares have their vertices in S?

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4 Three numbered tiles are arranged in a tray as shown:
`https://cdn.artofproblemsolving.com/attachments/d/0/d449364f92b7fae971fd348a82bafd25aa8ea`
`jpg`
Show that we cannot interchange the 1 and the 3 by a sequence of moves where we slide a tile to the adjacent vacant space.

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5 Show how to construct a line segment length $(a^4 + b^4)^{1/4}$ given segments lengths a and b.

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6 Five spheres of radius r are inside a right circular cone. Four of the spheres lie on the base of the cone. Each touches two of the others and the sloping sides of the cone. The fifth sphere touches each of the other four and also the sloping sides of the cone. Find the volume of the cone.
