Brazil National Olympiad 1983

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by parmenides51

1 Show that there are only finitely many solutions to $1/a+1/b+1/c = 1/1983$ in positive integers.

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2 An equilateral triangle ABC has side a. A square is constructed on the outside of each side of the triangle. A right regular pyramid with sloping side a is placed on each square. These pyramids are rotated about the sides of the triangle so that the apex of each pyramid comes to a common point above the triangle. Show that when this has been done, the other vertices of the bases of the pyramids (apart from the vertices of the triangle) form a regular hexagon.

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3 Show that $1 + 1/2 + 1/3 + ... + 1/n$ is not an integer for $n > 1$.

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4 Show that it is possible to color each point of a circle red or blue so that no right-angled triangle inscribed in the circle has its vertices all the same color.

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5 Show that $1 \leq n^{1/n} \leq 2$ for all positive integers n.
Find the smallest k such that $1 \leq n^{1/n} \leq k$ for all positive integers n.

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6 Show that the maximum number of spheres of radius 1 that can be placed touching a fixed sphere of radius 1 so that no pair of spheres has an interior point in common is between 12 and 14.

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