Brazil National Olympiad 1984

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by parmenides51

1 Find all solutions in positive integers to $(n+1)^k - 1 = n!$

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2 Each day 289 students are divided into 17 groups of 17. No two students are ever in the same group more than once. What is the largest number of days that this can be done?

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3 Given a regular dodecahedron of side a. Take two pairs of opposite faces: E, E' and F, F'. For the pair E, E' take the line joining the centers of the faces and take points A and C on the line each a distance m outside one of the faces. Similarly, take B and D on the line joining the centers of F, F' each a distance m outside one of the faces. Show that $ABCD$ is a rectangle and find the ratio of its side lengths.

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4 ABC is a triangle with $\angle A = 90^o$. For a point D on the side BC, the feet of the perpendiculars to AB and AC are E and$F$. For which point D is EF a minimum?

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5 $ABCD$ is any convex quadrilateral. Squares center E, F, G, H are constructed on the outside of the edges AB, BC, CD and DA respectively. Show that EG and FH are equal and perpendicular.

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6 There is a piece on each square of the solitaire board shown except for the central square. A move can be made when there are three adjacent squares in a horizontal or vertical line with two adjacent squares occupied and the third square vacant. The move is to remove the two pieces from the occupied squares and to place a piece on the third square. (One can regard one of the pieces as hopping over the other and taking it.) Is it possible to end up with a single piece on the board, on the square marked X?
