Brazil National Olympiad 1985

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by parmenides51

1 a, b, c, d are integers with $ad \neq bc$. Show that $1/((ax+b)(cx+d))$ can be written in the form $r/(ax+b)+s/(cx+d)$. Find the sum $1/1 \cdot 4+1/4 \cdot 7+1/7 \cdot 10+...+1/2998 \cdot 3001$.

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2 Given n points in the plane, show that we can always find three which give an angle $\leq \pi/n$.

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3 A convex quadrilateral is inscribed in a circle of radius 1. Show that the its perimeter less the sum of its two diagonals lies between 0 and 2.

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4 a, b, c, d are integers. Show that $x^2+ax+b=y^2+cy+d$ has infinitely many integer solutions iff $a^2-4b=c^2-4d$.

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5 A, B are reals. Find a necessary and sufficient condition for $Ax+B[x]=Ay+B[y]$ to have no solutions except $x=y$.
