Brazil National Olympiad 1986

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1 A ball moves endlessly on a circular billiard table. When it hits the edge it is reflected. Show that if it passes through a point on the table three times, then it passes through it infinitely many times.

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2 Find the number of ways that a positive integer n can be represented as a sum of one or more consecutive positive integers.

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3 The Poincare plane is a half-plane bounded by a line R. The lines are taken to be
(1) the half-lines perpendicular to R, and
(2) the semicircles with center on R.
Show that given any line L and any point P not on L, there are infinitely many lines through P which do not intersect L. Show that if ABC is a triangle, then the sum of its angles lies in the interval $(0, \pi)$.

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4 Find all 10 digit numbers $a_0a_1...a_9$ such that for each k, a_k is the number of times that the digit k appears in the number.

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5 A number is written in each square of a chessboard, so that each number not on the border is the mean of the 4 neighboring numbers. Show that if the largest number is N, then there is a number equal to N in the border squares.
