Brazil National Olympiad 1988

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1 Find all primes which are sum of two primes and difference of two primes.

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2 Show that, among all triangles whose vertices are at distances 3,5,7 respectively from a given point P, the ones with largest area have P as orthocenter.

(*You can suppose, without demonstration, the existence of a triangle with maximal area in this question.*)

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3 Find all functions $f:\mathbb{N}^*\rightarrow\mathbb{N}$ such that

- $f(x\cdot y)=f(x)+f(y)$
- $f(30)=0$
- $f(x)=0$ always when the units digit of x is 7

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4 Two triangles are circumscribed to a circumference. Show that if a circumference containing five of their vertices exists, then it will contain the sixth vertex too.

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5 A figure on a computer screen shows n points on a sphere, no four coplanar. Some pairs of points are joined by segments. Each segment is colored red or blue. For each point there is a key that switches the colors of all segments with that point as endpoint. For every three points there is a sequence of key presses that makes the three segments between them red. Show that it is possible to make all the segments on the screen red. Find the smallest number of key presses that can turn all the segments red, starting from the worst case.
