

Brazil National Olympiad 1989

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- 1 The sides of a triangle T , with vertices $(0, 0)$, $(3, 0)$ and $(0, 3)$ are mirrors.
Show that one of the images of the triangle T_1 with vertices $(0, 0)$, $(0, 1)$ and $(2, 0)$ is the triangle with vertices $(24, 36)$, $(24, 37)$ and $(26, 36)$.

- 2 Let k a positive integer number such that $\frac{k(k+1)}{3}$ is a perfect square. Show that $\frac{k}{3}$ and $k+1$ are both perfect squares.

- 3 A function f , defined for the set of integers, is such that $f(x) = x - 10$ if $x > 100$ and $f(x) = f(f(x + 11))$ if $x \leq 100$.
Determine, justifying your answer, the set of all possible values for f .

- 4 A game is played by two contestants A and B, each one having ten chips numbered from 1 to 10. The board of game consists of two numbered rows, from 1 to 1492 on the first row and from 1 to 1989 on the second.
At the n -th turn, $n = 1, 2, \dots, 10$, A puts his chip numbered n in any empty cell, and B puts his chip numbered n in any empty cell on the row not containing the chip numbered n from A.
B wins the game if, after the 10th turn, both rows show the numbers of the chips in the same relative order. Otherwise, A wins.

- Which player has a winning strategy?
- Suppose now both players has k chips numbered 1 to k . Which player has a winning strategy?
- Suppose further the rows are the set \mathbb{Q} of rationals and the set \mathbb{Z} of integers. Which player has a winning strategy?

- 5 A tetrahedron is such that the center of the its circumscribed sphere is inside the tetrahedron.
Show that at least one of its edges has a size larger than or equal to the size of the edge of a regular tetrahedron inscribed in this same sphere.