Spain Mathematical Olympiad 1994

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–	Day 1
1	Prove that if an arithmetic progression contains a perfect square, then it contains infinitely many perfect squares.
2	Let $Oxyz$ be a trihedron whose edges x, y, z are mutually perpendicular. Let C be the point on the ray z with $OC = c$. Points P and Q vary on the rays x and y respectively in such a way that $OP + OQ = k$ is constant. For every P and Q, the circumcenter of the sphere through O, C, P, Q is denoted by W. Find the locus of the projection of W on the plane Oxy. Also find the locus of points W.
3	A tourist office was investigating the numbers of sunny and rainy days in a year in each of six regions. The results are partly shown in the following table: Region , sunny or rainy , unclassified A 336 29 B 321 44 C 335 30 D 343 22 E 329 36 F 330 35 Looking at the detailed data, an officer observed that if one region is excluded, then the total number of rainy days in the other regions equals one third of the total number of sunny days in these regions. Determine which region is excluded.
–	Day 2
4	In a triangle ABC with $\angle A = 36^o$ and $AB = AC$, the bisector of the angle at C meets the oposite side at D. Compute the angles of $\triangle BCD$. Express the length of side BC in terms of the length b of side AC without using trigonometric functions.
5	Let 21 pieces, some white and some black, be placed on the squares of a 3×7 rectangle. Prove that there always exist four pieces of the same color standing at the vertices of a rectangle.
6	A convex n-gon is dissected into m triangles such that each side of each triangle is either a side of another triangle or a side of the polygon. Prove that $m + n$ is even. Find the number of sides of the triangles inside the square and the number of vertices inside the square in terms of m and n.