Spain Mathematical Olympiad 1995

www.artofproblemsolving.com/community/c691165

by parmenides51, Konigsberg

–	Day 1
1	Consider all sets A of one hundred different natural numbers with the property that any three elements $a, b, c \in A$ (not necessarily different) are the sides of a non-obtuse triangle. Denote by $S(A)$ the sum of the perimeters of all such triangles. Compute the smallest possible value of $S(A)$.
2	Several paper-made disks (not necessarily equal) are put on the table so that there is some overlapping, but no disk is entirely inside another. The parts that overlap are cut off and removed. Show that the remaining parts cannot be assembled so as to form different disks.
3	A line through the centroid G of the triangle ABC intersects the side AB at P and the side AC at Q Show that $\frac{PB}{PA} \cdot \frac{QC}{QA} \leq \frac{1}{4}$. Sorry for Triple-Posting. If possible, please merge the solutions to one document. I think there was an error because it may have automatically triple-posted.
–	Day 2
4	Given a prime number p, find all integer solutions of $p(x+y) = xy$.
5	Prove that if the equations $x^3 + mx - n = 0$ $nx^3 - 2m^2x^2 - 5mnx - 2m^3 - n^2 = 0$ have one root in common ($n \neq 0$), then the first equation has two equal roots, and find the roots of the equations in terms of n.
6	Let C be a variable interior point of a fixed segment AB. Equilateral triangles ACB' and CBA' are constructed on the same side and ABC' on the other side of the line AB. (a) Prove that the lines AA', BB' , and CC' meet at some point P. (b) Find the locus of P as C varies. (c) Prove that the centers A'', B'', C'' of the three triangles form an equilateral triangle. (d) Prove that A'', B'', C'' , and P lie on a circle.